On rigidity of spacetimes with a compact Cauchy horizon

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There is a great variety of solutions of Einstein's equation which contain a smooth compact Cauchy horizon. However, all of these spacetimes can be considered as being generalizations of the Taub-NUT solution. In these spacetimes, the Cauchy horizon separates the globally hyperbolic region from a region which contains closed timelike curves. It is a common feature of all of these spacetimes that they admit a 'horizon compatible' Killing vector field which is spacelike in the globally hyperbolic region, null on the horizon while the Killing orbits are closed timelike curves in the chronology violating region.

Since in all particular examples there exists a Killing vector field it is reasonable to ask: To what extent the field equations can guarantee the existence of such a Killing vector field? Clearly, this question has a direct relation to the validity of the strong cosmic censor hypothesis. This hypothesis of Penrose claims that all the 'physically reasonable' generic spacetimes are globally hyperbolic, i.e. they are inextendible Cauchy developments of 'maximal' initial data specifications. In this respect, if one could show that the existence of a compact Cauchy horizon is always associated with the presence of a symmetry one would have an indirect argument supporting the validity of Penrose's cosmic censor hypothesis.

Actually, a fundamental result of this type was given by Moncrief and Isenberg [3, 2]. They considered analytic electrovac spacetimes possessing a compact Cauchy horizon generated by closed null geodesics. As shown by the authors there must exist then a Killing vector field which is tangential to the null geodesic generators of the horizon and spacelike on the Cauchy development side. This result of Moncrief and Isenberg had been established by the mid of 80's. It has remained, however, an interesting open problem whether the analyticity assumption can be replaced by a less restrictive differentiability condition. It is worth emphasizing that, since the use of the analyticity assumption is incompatible with the concept of causality e.g. in the framework of initial value problem, this issue has not only of pure mathematical interest.

To construct a 'candidate' Killing vector field in the smooth setting the relation

$$\nabla^e \nabla_e K^a + R^a{}_d K^d = 0 \tag{1}$$

can be used, which is known to be satisfied by any Killing vector field. This is a wave equation for K^a suggesting that our approach should be based on an initial value problem. Since the horizon is null, the use of the null characteristic initial value problem seems to be appropriate where the initial data are specified on a pair of null hypersurfaces which intersect on a smooth spacelike 2-surface.

This means that the Cauchy horizon, by itself, does not comprise a suitable initial data surface for (1). In fact, the main difficulty one has to face in generalizing the Isenberg-Moncrief theorem to the smooth setting is that suitably detailed information about the spacetime metric and the electromagnetic field is known only on the horizon. To be able to use the characteristic initial value

problem an additional null hypersurface, on which (at least) the Lie derivative of the spacetime metric vanishes, was needed to be found.

By making use of a straightforward generalization of the techniques applied in Refs.[4, 5], such a null hypersurface can be constructed whenever the original Cauchy horizon is non-degenerate[1]. In addition, the Lie derivatives of the spacetime metric and the electromagnetic field with respect to a suitably chosen vector field, k^a, vanish throughout the resulting bifurcate horizon. This horizon provides an appropriate initial data surface. Upon having this surface, it remains to show that the unique solution K^a of (1), corresponding to initial data compatible with k^a, will be a Killing vector field in the associated Cauchy development. It follows, however, that, whenever K^a fulfills (1) but is otherwise an arbitrary vector field, the Lie derivatives of the spacetime geometry and the electromagnetic field variables satisfy a coupled system of linear and homogeneous wave equations in these basic variables[1, 7]. These type of equations are known to possess unique solutions, hence, they possess the identically zero solution for vanishing initial data.

These results were also found to work in the cases of Klein-Gordon, Higgs, Yang-Mills, Yang-Mills-dilaton and Yang-Mills-Higgs fields in Einstein's theory of gravity[6, 7].[1] The following statement, that applies to all of these general Einstein-matter systems, sums up the argument outlined above:

Theorem: *Consider a smooth spacetime with a compact Cauchy horizon generated by closed null geodesics. Suppose that the horizon is non-degenerate. Then there exists a smooth Killing vector field k^a in a sufficiently small neighbourhood of the horizon in the Cauchy development region. The horizon is a Killing horizon with respect to k^a, while this Killing vector field is spacelike off the horizon. The matter fields are also invariant.*

According to this result the presence of a compact non-degenerate Cauchy horizon, ruled by closed null geodesics, is really an artifact of a spacetime symmetry. This, in turn, supports the validity of the cosmic censor hypothesis by demonstrating the non-genericness of spacetimes possessing such a compact Cauchy horizon.

Acknowledgments

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References

[1] H. Friedrich, I. Rácz and R.M. Wald: Commun. Math. Phys. **204**, 691 (1999)

[2] J. Isenberg and V. Moncrief: J. Math. Phys. **26**, 1024 (1985)

[3] V. Moncrief and J. Isenberg: Commun. Math. Phys. **98**, 387 (1983)

[4] I. Rácz and R.M. Wald: Class. Quant. Grav. **9**, 2643 (1992)

[5] I. Rácz and R.M. Wald: Class. Quant. Grav. **13**, 539 (1996)

[6] I. Rácz: Class. Quant. Grav. **17**, 153 (2000)

[7] I. Rácz: Class.Quant.Grav. **16**, 1695 (1999)

[1] It is worth emphasizing that in the analytic setting the underlying techniques provide a generalization not only of the result of Moncrief and Isenberg but that of the Hawking's black hole rigidity theorem, as well.